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Peak load minimization in smart grid by optimal coordinated ON–OFF scheduling of air conditioning compressors

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ABSTRACT

We address the problem of minimizing the peak load by optimal coordinated ON–OFF scheduling of the compressors of air conditioners (ACs) connected in a smart grid. For this purpose, we consider a simplified model of power consumption profile, i.e., on-time and off-time durations and power consumption values of a split type AC. We model the necessary constraints and formulate an optimization problem to minimize the peak load by optimal coordinated ON–OFF scheduling of the AC compressors. The optimization problem is found to be a complex mixed integer linear programming problem. We optimally solve the problem for a small number of ACs by using an optimization tool. Unfortunately, due to the computational complexity, the tool cannot solve the problem for a large number of ACs. For a large number of ACs, we develop a heuristic algorithm to solve the problem. Using the optimization tool and the heuristic algorithm, we determine the peak load, load variance, and energy consumption in operating a number of ACs and compare them with the results obtained for a traditional non-coordinated AC operation. We find that both the optimal and heuristic solution approaches significantly reduce the peak load and load variance with some increment of energy consumption. Further, the computation time of the scheduling of the AC compressors of an air conditioning system under the heuristic algorithm is found to be significantly less compared to the time bound on scheduling computation of the AC compressors even when the number of ACs in the system is large.

1. Introduction

Smart grid (SG) is a rejuvenated form of the existing power grid system for electricity generation, transmission, distribution and control with energy savings, reduced cost and increased reliability, security, safety, quality of service and transparency [1]. Among the smart management, smart protection and energy, and smart information and communication sub-systems of SG, smart management sub-system (SMSS) plays a vital role in achieving the goals of SG [2]. Demand side management (DSM) is a salient feature of the SMSS that provides emphasis towards advanced functionalities in electricity market control, infrastructure redesign, demand profile reshaping, and reduction of peak load demand as well as overall cost [3]. Reduction of peak load as well as variation of load is very important for power system stability. Load scheduling is considered as the key technique in reducing peak load as well as load variation of a grid [4].

Electrical loads are two types: time flexible and non-flexible. Time flexible loads, e.g., charging of electrical vehicles, washers, dryers are usually scheduled during the off-peak hours of a day to satisfy specific objective [5–7]. Various load management and aggregation methods for time flexible loads are presented in [8–10]. Scheduling the time non-flexible loads, e.g., light, fan, air conditioner (AC), refrigerator etc., is very critical since the appliances and the elements of this kind of load cannot be turned off during operation. Among the time non-flexible loads, scheduling of air conditioning load has become a prime subject to reduce peak load [11] since a significant portion of total electricity consumption is consumed by the ACs in residential dwellings and industries. Based on the availability and usage, both the inverting and non-inverting ACs are widely used. However, non-inverting ACs are less expensive than the inverting ACs. If the usage per day is less, a non-inverting AC is cost effective compared to an inverting AC. Moreover, maintenance and repair of a non-inverting AC is easier as the operation mechanism of this type of ACs is simple and knowledgeable technicians and spare parts are available. On the other hand, maintenance and repair of an inverting AC is difficult as the operation mechanism of this type of ACs is complex and knowledgeable technicians and spare parts are not available. Tremendous research interests are observed on integrating the both types of ACs to SG. However, we consider the non-inverting ACs for this research as they are cheap and their operation and maintenance is easier.

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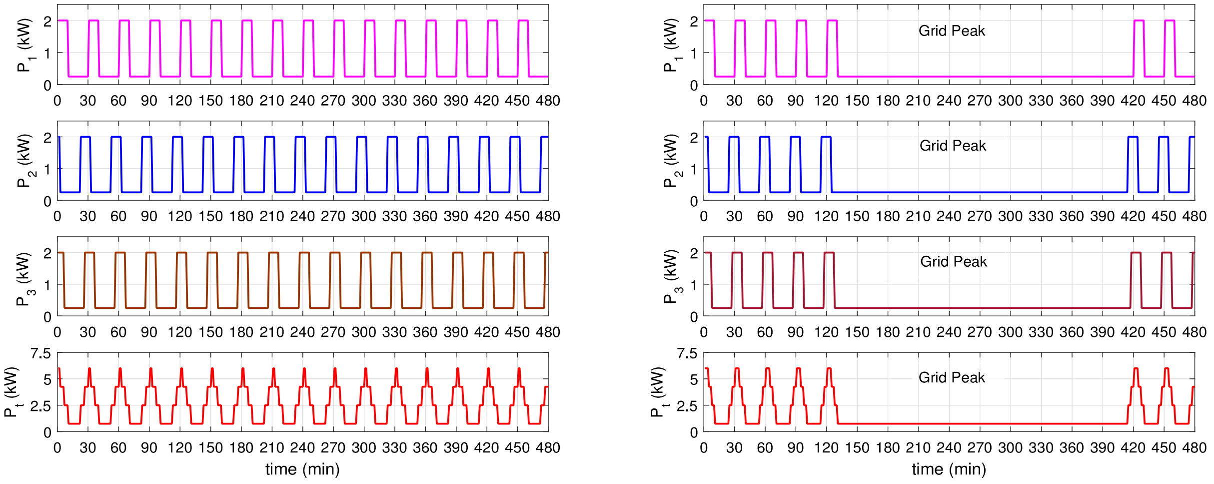
There are a significant number of research works on scheduling the load of non-inverting ACs connected in SG with different objectives [12–27]. These research works can be classified into two categories: (i) satisfying the objective by turning off ACs for a long time period [12–15] and (ii) satisfying the objective by increasing the temperature set point or by reducing power consumption of ACs [16–27]. In [12], the authors propose a direct load control to reduce peak load and production cost by turning off all the ACs for a cycle at the peak loads which is executed by a dynamic programming. In [13], the authors apply cycling control to reduce peak load by turning OFF the AC loads during peak load time keeping the other non-flexible loads ON. The authors in [14] consider the power profile of various loads including ACs and propose a heuristic algorithm to control the activation or deactivation of a set of loads for peak shaving. In [15], the authors provide an optimization based solution to keep OFF the ACs when electricity price is high and to keep ON the ACs when the electricity price is low. Unfortunately, under the scheduling methods in [12–15], the consumers must sacrifice their comfort during the peak load for turning off the ACs which are not enough intelligent solutions.

Among the research works satisfying the objective by increasing the temperature set point or by reducing power consumption of ACs, the authors in [16] address the load scheduling, shifting and parameter configuration problem for the both time flexible and non-flexible household loads with one hour time slot by taking the overall costs and climatic comfort level into account. The authors in [17] address the scheduling of AC loads integrating with wind power to minimize the total system operation costs considering the thermal comfort of customers and indoor temperature variation with time. In [18], the authors propose to adjust ON–OFF durations of AC compressors based on environment temperature to maximize energy efficiency. In [19], the authors address the peak load reduction problem using load scheduling by solving a mixed integer linear programming (MILP) problem with one hour time slot where the flexible loads are shifted and the AC loads are run with an adjusted thermostat set points during the peak load. In [20], the temperature set points of ACs are controlled within the comfort limits of the consumers to minimize power consumption under dynamic pricing. In [21], the authors propose a programmable communicating thermostat to adjust temperature set points of ACs based on environment temperature to reduce the peak demand. In [22], the authors propose an AC scheduling scheme based on real time pricing such that the users reduce power consumption by increasing the temperature set points within comfort levels. The authors in [23] consider the AC loads as reducible up to a certain limit and present an optimization algorithm to schedule the household appliances for minimizing monthly bill. In [24], the authors consider the AC load as power elastic and provide an optimal scheduling algorithm to minimize the electricity consumption cost. The authors in [25] propose a robust optimization based load scheduling to minimize total power consumption for the loads with uncertain parameters. In [26], the authors present a computationally inexpensive, dynamic, and retrofit-deployable control strategy for peak load reduction and load shaping by controlling load thermostatically based on priority. In [27], the proposed control mechanism selects an operating schedule for the ACs that maintains a temperature set point subject to a constraint on the number of AC units that may operate simultaneously. From the operation point of view, power consumption of an AC can be reduced by increasing the temperature set point. Thus, both the strategies, i.e., increasing the temperature set point and reducing the power consumption are the same. However, it will result in the variation of room temperature and hence, the variation of the temperature set point at the different times of a day may provide discomfort to the consumers and there may also be health related issues. However, these researches assume that the temperature variation due to the increasing of the temperature set point or reducing the power consumption remains in a comfort band of the consumer.

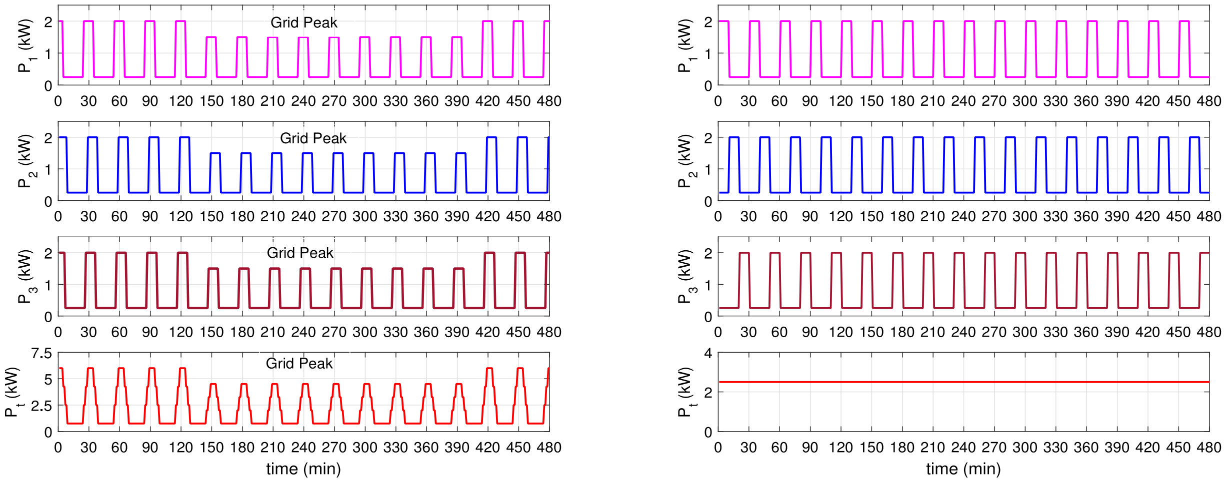
In this research, we assume that the temperature set point set by a user should be kept fixed to provide maximum level of comfort. This research work started with the question: *How the loads of ACs should be scheduled such that temperature set point remains fixed and the peak load can be minimized?* To address the problem, we find a way by optimal coordinated scheduling the ON–OFF timing of the AC compressors. The optimal solution will provide a coordinated ON–OFF timing of the compressors such that the peak load is minimized. Note that the power profile of a non-inverting AC can be approximated as an ON–OFF process [28]. The on–off time durations and power consumption values of a non-inverting AC depend on numerous factors such as coefficient of performance, ambient temperature, cooling capacity, temperature set point, *dead-band* of the compressor thermostat and so on [29]. The *dead-band* is a range of temperature around the temperature set-point in which the compressor of an AC operates. The authors in [28] consider homogeneous compressors in a refrigerated warehouse and propose to turn ON the compressors one after another with a delay equals to on-time duration and then the compressors continue the ON and OFF operation as usual. Unfortunately, the ACs in an air conditioning system are not homogeneous, thus, the method proposed in [28] is not applicable. To the best of our knowledge, peak load reduction is not yet addressed by using the intelligent and coordinated ON–OFF scheduling of the compressors of ACs. The motivation of this work is to reduce peak load by optimal coordinated ON–OFF scheduling of the compressors of ACs. In Fig. 1, we demonstrate the operating principles of traditional, existing and *proposed optimal coordinated scheduling* techniques considering three ACs for the operation of 8 h, where P_1, P_2 and P_3 are the power consumption of three ACs, respectively at different times of 8 h and P_t is the total power consumption for the three ACs. We assume that the peak load at the connected grid is from 130 min to 410 min. For the traditional technique, the compressors of the ACs turn ON and OFF without any coordination, some times the compressors of the three ACs become ON simultaneously and hence, the peak load is significantly high. In the existing work of type-1, the ACs are kept OFF during the grid peak power durations by optimal load scheduling. On the other hand, the power of the ACs reduced during the grid peak power by increasing the temperature set points in the existing work of type-2. In the proposed optimal coordinated ON–OFF scheduling technique, the compressors of the ACs turn ON and OFF by coordinated optimal scheduling and hence, peak power is reduced significantly. For the proposed optimal coordinated ON–OFF scheduling, the power reduces not only during the grid peak but also during grid off-peak. Note that the proposed coordinated on–off scheduling can be used with increment of temperature set point or decrement of power consumption of each of the ACs during the grid peak by maintaining a comfort temperature band of the consumer. In this research, we focus on the coordinated ON–OFF scheduling of compressors of ACs without considering the non-flexible loads as it is completely a new idea in the research of load scheduling of AC system. Our main objective is to provide insight on how much the peak load can be reduced by optimal operation of the ACs without considering the non-flexible loads.

We consider an air conditioning system with non-inverting ACs of different capacities. For an AC unit, we use the simple models of on-time and off-time durations and power consumption values developed in [29]. We assume that there is a range of dead-band temperature for AC operation and the consumers set temperature set points of the ACs randomly. The ACs are installed



(a) Traditional

(b) Existing work type-1



(c) Existing work type-2

(d) Proposed Optimal Coordinated ON-OFF Scheduling

Fig. 1. Typical examples of operation of ACs under various methods.

with Internet of things (IoT) transceivers for communicating with a server. The time is assumed to be slotted into a time scale of minute. The server computes the ON–OFF scheduling of the compressors and sends to the ACs for operation. The contributions in this paper are as follows:

- We introduce the notion of coordinated ON–OFF scheduling of compressors to minimize the peak load and model the necessary constraints for coordinated ON–OFF scheduling of the compressors so that the temperature set points of the ACs remain unchanged and the dead-band temperatures of the ACs keep within a predefined bound. By modelling the other required constraints, we formulate an optimization problem to determine the optimal coordinated ON–OFF scheduling of the AC compressors to minimize the peak load. The optimization problem is found to be a complex MILP problem.
- We optimally solve the problem by using the optimization tool CPLEX for a small number of ACs. For a large number of ACs, the tool is incapable to solve the problem within time limit.
- For a large number of ACs, we propose a heuristic algorithm to solve the optimization problem. The heuristic algorithm is found to be very effective in solving the problem within a short time with an ample good solution.
- We compare the peak load, load variance and energy consumption obtained by the optimal and the heuristic solution techniques with those obtained for a traditional non-coordinated AC operations. We demonstrate that the peak load and load variance can be reduced significantly by coordinated ON–OFF scheduling of AC compressors with some increment of energy consumption.

The rest of the paper is organized as follows. In Section 2, we describe the air conditioning system and formulate the optimization problem. In Section 3, we propose solution approaches for the optimization problem including the heuristic algorithm. In Section 4, we present numerical results and evaluate the performance of the proposed technique. Section 5 concludes the paper.

2. System description and problem formulation

In this section, we first present the electrical power consumption profile model, i.e., on-time and off-time durations and power consumption values for a split type AC. Then, we describe the operation of the air conditioning system consisting of a number of

ACs. Finally, we formulate the optimization problem for optimal coordinated ON–OFF scheduling of the compressors of the ACs of the air conditioning system.

2.1. Models of on-time and off-time durations and power consumption values for an AC unit

Before presenting the models of on-time duration, off-time duration and power consumption, let us briefly explain the operation of an AC. An AC uses a thermostat with a temperature sensor to control the room temperature. The temperature that the user sets through remote control is called the *set-point* temperature which is the target room temperature. The *dead-band* is a range of temperature around the set-point in which the room temperature varies by turning on and off the AC compressor. The operating time of the compressor that is needed to decrease the room temperature to the lower dead-band temperature from the upper dead-band is known as *on-time*. On the other hand, when the compressor is not operating, the room temperature increases and the time required to increase the room temperature to the upper dead-band temperature from the lower dead-band temperature is known as *off-time*.

Let on-time, off-time and cycle durations are denoted by T_{ON}, T_{OFF} and T, respectively. It is well know that $T = T_{ON} + T_{OFF}$. Let $H^r(t)$ be the room temperature at time t, M_{air} be the mass of air inside the room, C_p be the specific heat of air at constant pressure, R_{eq} be the equivalent envelop resistance of the room, and H_{out} be the ambient temperature. Using the classic thermodynamics equations mentioned in [30], one can find that,

$$M_{air}R_{eq}C_p\frac{dH^r(t)}{dt} = P_hR_{eq} + H_{out} - H^r(t). \tag{1}$$

where, $P_h = \eta P_{input}$ if the co-efficient of performance be η and the power consumption of the AC be P_{input}. Let H_{set} be the set point by the user and ΔH be the dead-band temperature. The compressor turns on whenever the room temperature is greater or equal to $H_{set} + \frac{\Delta H}{2}$. During the on-time, the room temperature decreases from $H_{set} + \frac{\Delta H}{2}$ to $H_{set} - \frac{\Delta H}{2}$. So, integrating dt in (1) over this temperature range, on-time duration, T_{ON} can be found as

$$T_{ON} = M_{air}R_{eq}C_p \ln\frac{P_h^{on}R_{eq} + H_{out} - (H_{set} + \frac{\Delta H}{2})}{P_h^{on}R_{eq} + H_{out} - (H_{set} - \frac{\Delta H}{2})} \tag{2}$$

where, P_h^{on} is the heat exchanged between the AC and the room during on-time. Since 3517 watts is equivalent to 1 *ton* refrigeration and $P_h^{on} < 0$, $P_h^{on} = -3517C$, where C is the capacity of AC in *ton*. Further, $M_{air} = \rho v$, where ρ and v are the density of air and volume of the room respectively. Again, $v = Ah$, where A and h are the area and height of the room. So, $M_{air} = Ah\rho$. Thus, the on-time can be found as

$$T_{ON} = \alpha AR_{eq}\ln\frac{-3517CR_{eq} + H_{out} - (H_{set} + \frac{\Delta H}{2})}{-3517CR_{eq} + H_{out} - (H_{set} - \frac{\Delta H}{2})} \tag{3}$$

where, $\alpha = hC_p\rho$ is a co-efficient of the model. Let $\beta = \frac{-\frac{\Delta H}{2}}{-3517CR_{eq}+H_{out}-H_{set}}$. Using the value of β in (3), T_{ON} can be written as $T_{ON} = \alpha AR_{eq}\ln\frac{1+\beta}{1-\beta}$. Since $\beta \ll 1$, T_{ON} can be approximated as $T_{ON} \approx 2\alpha AR_{eq}\beta$. Finally, the simplified expression of T_{ON} can be obtained as

$$T_{ON} \approx \frac{\alpha AR_{eq}\Delta H}{3517CR_{eq} - H_{out} + H_{set}}. \tag{4}$$

During off-time, the room temperature increases from $H_{set} - \frac{\Delta H}{2}$ to $H_{set} + \frac{\Delta H}{2}$. So, by integrating dt in (1) over this temperature range, T_{OFF} can be obtained as,

$$T_{OFF} = \alpha AR_{eq}\ln\frac{P_h^{off}R_{eq} + H_{out} - (H_{set} - \frac{\Delta H}{2})}{P_h^{off}R_{eq} + H_{out} - (H_{set} + \frac{\Delta H}{2})} \tag{5}$$

where, $P_h^{off} = \eta P_{OFF}$ is the heat exchange between the fan of the AC and the room with off-time power consumption P_{OFF}. Using similar approximation of on-time modelling, the simplified expression of T_{OFF} can be obtained as,

$$T_{OFF} \approx \frac{\alpha AR_{eq}\Delta H}{P_h^{off}R_{eq} + H_{out} - H_{set}}. \tag{6}$$

Note that the on-time and off-time of an AC depend on not only the ambient temperature but also on the solar gain and internal gain including the building/zone dynamics. For simplicity of analysis, we consider only the ambient temperature. It should be mentioned that the optimization models described later use the on-times and off-times of the ACs as variables. Thus, the optimization models will be applicable even if the models of on-time and off-time use the ambient temperature, solar gain, internal gain and the building/zone dynamics etc. as variables.

Since only the fan of the AC operates during the off-time of the compressor, the off-time power consumption, P_{OFF} is equal to the power consumption of the fan of the AC P_{fan} and $P_h^{off} = \eta P_{fan}$. Thus, power consumption values during on-time and off-time, i.e., P_{ON} and P_{OFF} can be written as

$$P_{ON} = \frac{3517C}{\eta} \tag{7}$$

and

$$P_{OFF} = P_{fan}. \tag{8}$$

2.2. Air conditioning system

We consider that there are N ACs in the air conditioning system. The set of the ACs is denoted by $\mathcal{N}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$. We assume that all the ACs are installed with IoT transceivers which communicate with a server through WiFi or cellular base station. Let the area of the room, the capacity, and the temperature set point of the AC $n \in \mathcal{N}$ be A_n, C_n and H_{set}^n, respectively. We assume that consumers receive 100% comfort if the temperature set points of the ACs are kept fixed to their set points. Thus, temperature set points of the ACs are kept fixed to the set points. The environment temperature is H_{out} during the operation time. The IoT transceivers send the area, capacity and temperature information to the server if they change. The server computes the schedule of ON–OFF times of the compressors of the ACs for a time duration T_s by solving the optimization problem and sends the schedules to the IoT transceivers. During the operation time T_s, each AC turns on and off its compressor according to the ON–OFF schedule. We assume that the time duration T_s is short, e.g., 90 min and the initial state of the building envelope, the outdoor temperature and the coefficient of performance remain unchanged during the short time duration T_s. The server periodically computes the ON–OFF schedules with time interval T_s and sends to the ACs. We assume that the ON–OFF scheduling of the ACs maintains a lower bound and an upper bound on dead-band temperature. In the following sub-section, optimization problem is formulated to solve the On–Off scheduling of the compressors for a time duration T_s.

2.3. Problem formulation

In this sub-section, we formulate the optimization problem for minimizing the peak load with optimal coordinated scheduling of the compressors of the ACs. Let the lower and upper bounds of dead-band temperature of all the ACs be ΔH_l and ΔH_u, respectively. For an AC $n \in \mathcal{N}$, the parameters ΔH_l, ΔH_u, A_n, C_n and H_{set}^n will provide a lower bound and an upper bound on on-time and off-time from (4) and (6). Let T_{ON-max}^n (resp. $T_{OFF-max}^n$) and T_{ON-min}^n

(resp. $T^n_{OFF-min}$) be the bounds on maximum and minimum on-time (resp. off-time), respectively for the AC $n \in \mathcal{N}$. Let the time duration T_s is slotted into time slots and the duration of each slot is one minute. The set of the time slots is presented as $\mathcal{T}_s = \{1, 2, \dots, T_s\}$. For simplicity, we assume that T^n_{ON-max}, $T^n_{OFF-max}$, T^n_{ON-min} and $T^n_{OFF-min}$ are integer multiple of a time slot duration. Let P^n_{ON} and P^n_{OFF} be the on-time and off-time power consumption of the AC $n \in \mathcal{N}$. Let $x_{n,t}$, $y_{n,t}$, and $z_{n,t}$ be the binary decision variables for all $n \in \mathcal{N}, t \in \mathcal{T}_s$. The binary variable $x_{n,t} = 1$ if the compressor of the AC n is ON during the time slot t and $x_{n,t} = 0$ otherwise. The binary variable $y_{n,t} = 1$ if the compressor of the AC n turns ON at time slot t from OFF condition and $y_{n,t} = 0$ otherwise. The binary variable $z_{n,t} = 1$ if the compressor of the AC n turns OFF at time slot t from ON condition and $z_{n,t} = 0$ otherwise. The matrices for the $x_{n,t}$, $y_{n,t}$ and $z_{n,t}$ variables are denoted by $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$, respectively. The total power consumption at time slot $t \in \mathcal{T}_s$, P_t can be found as

$$P_t = \sum_{n\in\mathcal{N}} x_{n,t}P^n_{ON} + \sum_{n\in\mathcal{N}}(1 - x_{n,t})P^n_{OFF}. \tag{9}$$

Denote by $\mathbf{P}$ the vector of the variables P_t. Let P be the maximum value of the P_t variables. The objective of the optimal ON–OFF scheduling is to find out the values of $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ to minimize P by maintaining the on-time and off-time bounds. Define the following sets of time-slots:

$\mathcal{T}^n_{t,T_{max}} = \{t, t+1, \dots, t + T^n_{ON-max} + T^n_{OFF-max} - 1\}$
$\mathcal{T}^n_{t,T_{min}} = \{t, t+1, \dots, t + T^n_{ON-min} + T^n_{OFF-min} - 1\}$
$\mathcal{T}^n_{t,T_{on-max}} = \{t, t+1, \dots, t + T^n_{ON-max} - 1\}$
$\mathcal{T}^n_{t,T_{on-min}} = \{t, t+1, \dots, t + T^n_{ON-min} - 1\}$
$\mathcal{T}^n_{t,T_{off-max}} = \{t, t+1, \dots, t + T^n_{OFF-max} - 1\}$
$\mathcal{T}^n_{t,T_{off-min}} = \{t, t+1, \dots, t + T^n_{OFF-min} - 1\}$
$\mathcal{T}^n_{s,T_{max}} = \{1, 2, \dots, T_s - T^n_{ON-max} - T^n_{OFF-max} + 1\}$
$\mathcal{T}^n_{s,T_{min}} = \{1, 2, \dots, T_s - T^n_{ON-min} - T^n_{OFF-min} + 1\}$
$\mathcal{T}^n_{s,T_{on-max}} = \{1, 2, \dots, T_s - T^n_{ON-max} + 1\}$
$\mathcal{T}^n_{s,T_{on-min}} = \{1, 2, \dots, T_s - T^n_{ON-min} + 1\}$
$\mathcal{T}^n_{s,T_{off-max}} = \{1, 2, \dots, T_s - T^n_{OFF-max} + 1\}$
$\mathcal{T}^n_{s,T_{off-min}} = \{1, 2, \dots, T_s - T^n_{OFF-min} + 1\}$

The optimization problem can be stated as follows.

Problem $\mathcal{P}_1$:

$$\min_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{P}} P \tag{10}$$

$$P_t \le P \qquad \forall t \in \mathcal{T}_s \tag{11}$$

$$P_t = \sum_{n\in\mathcal{N}} x_{n,t}P^n_{ON} + \sum_{n\in\mathcal{N}}(1 - x_{n,t})P^n_{OFF} \quad \forall t \in \mathcal{T}_s \tag{12}$$

$$\sum_{t'\in T^n_{t,T_{max}}} y_{n,t'} \ge 1 \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{max}} \tag{13}$$

$$\sum_{t'\in T^n_{t,T_{max}}} z_{n,t'} \ge 1 \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{max}} \tag{14}$$

$$\sum_{t'\in T^n_{t,T_{min}}} y_{n,t'} \le 1 \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{min}} \tag{15}$$

$$\sum_{t'\in T^n_{t,T_{min}}} z_{n,t'} \le 1 \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{min}} \tag{16}$$

$$\sum_{t'\in T^n_{t,T_{on-min}}} x_{n,t'} \ge y_{n,t}T^n_{ON-min} \qquad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{on-min}} \tag{17}$$

$$\sum_{t'\in T^n_{t,T_{off-min}}} (1 - x_{n,t'}) \ge z_{n,t}T^n_{OFF-min} \qquad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{off-min}} \tag{18}$$

$$\sum_{t'\in T^n_{t,T_{on-max}}} x_{n,t'} \le T^n_{ON-max} \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{on-max}} \tag{19}$$

$$\sum_{t'\in T^n_{t,T_{off-max}}} (1 - x_{n,t'}) \le T^n_{OFF-max} \qquad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{off-max}} \tag{20}$$

$$\sum_{t'\in T^n_{t,T_{max}}} x_{n,t'} \ge T^n_{ON-max} \quad \forall n \in \mathcal{N}, \forall t \in T^n_{s,T_{max}} \tag{21}$$

$$x_{n,t} = x_{n,t-1} + y_{n,t} - z_{n,t} \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}_s \setminus \{1\} \tag{22}$$

$$\mathbf{x}, \mathbf{y}, \mathbf{z} \in \{0, 1\} \tag{23}$$

$$0 \le \mathbf{P}, P \tag{24}$$

In the Problem $\mathcal{P}_1$, the objective function in (10) and the constraints in (11) ensure that the maximum power consumption for the duration T_s is minimized. The equality constraints in (12) represent the values of power consumption at different time slots. The constraints in (13) and (14) state that the compressor of an AC n must turn on at least once and must turn off at least once during the maximum cycle time ($T^n_{ON-max} + T^n_{OFF-max}$). The constraints in (15) and (16) state that the compressor of an AC n should not turn on more than once and should not turn off more than once during the minimum cycle time ($T^n_{ON-min} + T^n_{OFF-min}$). The constraints in (17) ensure that if the compressor of an AC n turns on it remains ON at least for the duration of the minimum on-time T^n_{ON-min}. Similarly, the constraints in (18) ensure that if the compressor of an AC n turns off it remains OFF at least for the duration of the minimum off-time $T^n_{OFF-min}$. The constraints in (19) and (20) are the bounds on the maximum on-time and the maximum off-time, respectively. The constraints mentioned above guarantee that on-time and off-time of a compressor will remain within the limit. However, they do not guarantee the dead-band bounds since the compressor of an AC may operate with minimum on-time and maximum off-time under the above constraints. Thus, to meet the dead-band bounds, in addition to the constraints mentioned above, the constraints in (21) are included which guarantee that the on-time of the compressor of an AC n is at least T^n_{ON-max} during the maximum cycle time ($T^n_{ON-max} + T^n_{OFF-max}$). Unfortunately, the constraints mentioned above do not guarantee the continuous ON and continuous OFF operation of an AC compressor, although they guarantee all the other requirements. Modelling the constraints to guarantee the continuous ON and continuous OFF operation of an AC compressor is very critical. However, the problem is solved by including the constraints in (21) which guarantee that the compressor of an AC n remains continuously ON until $z_{n,t}$ becomes 1 and it remains continuously OFF until $y_{n,t}$ becomes 1. The constraints in (23) define the integer variables and the constraints in (24) define the range of the variables.

The problem $\mathcal{P}_1$ is an MILP. The number of variables and the number of constraints are in the order of $\mathcal{O}(NT_s)$. Although the order of the number of variables and constraints are not too much, the optimization problem becomes very complex due to the structure of the constraints. The number of variables needs to be summed up for a constraint in (13)–(21) depends on the index of the constraint as the ranges of the variables $\mathcal{T}^n_{t,T_{max}}$, $\mathcal{T}^n_{t,T_{min}}$, $\mathcal{T}^n_{t,T_{on-max}}$, $\mathcal{T}^n_{t,T_{on-min}}$, $\mathcal{T}^n_{t,T_{off-max}}$ and $\mathcal{T}^n_{t,T_{off-min}}$ depend on the time instant t. These dependencies make the problem different and difficult. Moreover, the problem may be difficult to solve as huge searching or branching and bounding is required to find the feasible solution by satisfying the following conditions: (i) each compressor turns on at least once and turns off at least once during the maximum cycle time, (ii) each compressor does not turn on more than once and does not turn off more than once during the minimum cycle time, (iii) a compressor remains ON (resp. OFF) at least for the duration of the minimum on-time (resp. off-time) if it turns ON (resp. OFF), (iv) the on-time of a compressor is higher than or equal to the maximum on-time during the maximum cycle time, and (v) a compressor remains continuously ON (resp. OFF) until it turns off (resp. ON).

As described in Section 2.2, the optimization problems of two consecutive time periods are independent, i.e., the solution of the optimization problem of the next time period does not depend on the solution of the optimization problem of the current time period. As a result, the thermal constraints may not be satisfied by a few ACs during the transition time between two time periods. It is due to the fact that at the end of the current time period T_s, the compressors of a few ACs may remain OFF/ON and these OFF/ON periods are not considered during solving the optimization problem of the next time period T_s. So by solving the optimization problems of two consecutive time periods independently, the guarantee of thermal constraints for all the ACs may not be provided. Thus, specific adjustments are necessary to guarantee the thermal constraints for all the ACs during the transition time between two consecutive time periods as well as during the whole time duration of the operation of the ACs. The adjustments are nothing but the inclusion of the ON time and OFF time information of the compressors at the end of the current time period during solving the optimization problem for the next time period. Let w_n and w_n' be the number time slots that the compressor of the AC $n \in \mathcal{N}$ remain continuously ON and OFF, respectively, at the end of the current time period T_s. Let the set of ACs with their compressors ON (resp. OFF) at the end of the current time period is denoted by $\mathcal{N}_{ON}$ (resp. $\mathcal{N}_{OFF}$). Hence, the simplest solution is to set fixed values of ON time or OFF time for the ACs (i.e., set fixed values to the variables x of the optimization problem) at the starting of the next time period T_s so that each compressor remains ON/OFF at the starting of the next time period T_s to meet the thermal constraints. Thus, the following constraints can be included to the optimization problem $\mathcal{P}_1$ to meet the thermal constraints.

$$x_{n,t} = 1 \quad \forall n \in \mathcal{N}_{ON}, \forall t \in \{1, 2, \dots, T^n_{ON-min} - w_n\} \tag{25}$$

$$x_{n,t} = 0 \quad \forall n \in \mathcal{N}_{OFF}, \forall t \in \{1, 2, \dots, T^n_{OFF-min} - w_n'\} \tag{26}$$

In our research work, to avoid the complexity and to keep the problem simple as much as possible, we do not address all the practical issues, e.g., we do not consider the constraints in (25) and (26). Because the main objective of the work is to provide insight that the coordinated ON–OFF scheduling of the compressors of ACs can significantly reduce the peak load.

3. Solution approaches and algorithms

In this section, we propose two solution approaches for the MILP problem $\mathcal{P}_1$. The first approach is based on optimization tool and the second one is based on a heuristic algorithm. The solution approach based on optimization tool is called the optimal coordination algorithm.

3.1. Optimal coordination algorithm

Optimal coordination algorithm is executed by solving the optimization problem periodically by using an optimization tool. As mentioned earlier, the IoT transceivers of the ACs send all the environmental information to the server if they change and the server solves the problem periodically for a time duration T_s and sends the ON–OFF schedules to the IoT transceivers. We use CPLEX solver to solve the optimization problem which is available in NEOS server [31]. It requires a model file and a data file. The model file of the problem is generated using a mathematical programming language (AMPL) and the data file is generated using MATLAB. Unfortunately, due to the complexity of the problem, the solver can solve the problem only for a small number of ACs. Moreover, it takes longer time to solve the problem. Considering the practical applications with a larger number of ACs, we propose a heuristic algorithm to solve the optimization problem.

3.2. Solution by heuristic algorithm

The ACs are clustered into K types of clusters based on the values of T^n_{ON-max}, T^n_{ON-min}, $T^n_{OFF-max}$ and $T^n_{OFF-min}$ for all $n \in \mathcal{N}$. The set of the cluster types of the ACs is defined as $\mathcal{K} = \{1, 2, 3, \dots, K\}$. Let the set of the ACs of type $k \in \mathcal{K}$ be $\mathcal{N}_k$. Let T^k_{ON-max}, T^k_{ON-min}, $T^k_{OFF-max}$ and $T^k_{OFF-min}$ be the maximum on-time, minimum on-time, maximum off-time and minimum off-time, respectively, for each AC of the type $k \in \mathcal{K}$. The number of ACs of type $k \in \mathcal{K}$ that can be scheduled to be turned ON one after another without on-time overlapping during a time duration $T^k_{ON-max} + T^k_{OFF-max}$ is given as $n_k = \lfloor \frac{T^k_{ON-max} + T^k_{OFF-max}}{T^k_{ON-max}} \rfloor$. Again, the number of different sets of n_k number of ACs of the type k that can be scheduled one after another during a time duration $T^k_{ON-max} + T^k_{OFF-max}$ is given by $S_k = \lfloor \frac{|\mathcal{N}_k|}{n_k} \rfloor$. The block diagram of the proposed heuristic algorithm is presented in Fig. 2. In the initialization block, the variables are initialized and the parameters are loaded, and then ON–OFF scheduling of the ACs are solved by using four steps shown inside the dotted boxes. Define by $\mathcal{N}^r_k$ the set of ACs of type k remains to complete scheduling. Initially, we set $\mathcal{N}^r_k = \mathcal{N}_k, \forall k \in \mathcal{K}$.

3.2.1. Step-1

In Step-1, for each AC type $k \in \mathcal{K}$, compressors of n_k ACs are turned ON one after another without on-time overlapping with the maximum on-time. Those n_k compressors turn-off with the maximum off-time and the on–off process of the n_k ACs continues for the whole duration T_s. Each of the S_k different sets of n_k number of ACs are scheduled in similar fashion, i.e., the scheduling of $n_k S_k$ number of ACs are performed in Step-1. In the block diagram, time duration $(T^n_{ON-max} + T^n_{OFF-max})$ is denoted by T^n_{max} and the lth element in a set $\mathcal{Q}$ is denoted by $\mathcal{Q}[l]$. According to the scheduling in Step-1, for $s = 1, s \le S_k$, the compressor of ith AC in $\mathcal{N}_k$, $1 \le i \le n_k$, is first turned ON at time slot $((i-1)T^k_{ON-max} + 1)$ and one cycle time $(T^k_{ON-max} + T^k_{OFF-max})$ is completed by the first time turning ON of the n_k ACs. The compressors of these n_k ACs periodically turn-on and turn-off during the operation time T_s. Similarly, for each $s = 2, s = 3, \dots, s = S_k$, the compressors of n_k ACs are scheduled.

3.2.2. Step-2

It is clear that after completing the scheduling in Step-1, the number of remaining ACs to be scheduled for a type $k, \forall k \in \mathcal{K}$, is less than n_k. In Step-2, the remaining ACs of one type, say, k', $\mathcal{N}^r_{k'} \neq \{\phi\}$, with the highest ON time are scheduled one after another similar to the scheduling process in Step-1.

3.2.3. Step-3

In step-3, the remaining ACs are scheduled one by one by checking the power consumption at different time slots. We describe the scheduling method for an AC n' of a type k' below and the same method is applied for the others. The process starts from time slot $t = 1$. For the present value of t, the maximum power consumption value for each of the ON time windows $[t, t + T^{n'}_{ON-min} + dt_{k'} - 1]$, $dt_{k'} = \{0, 1, 2, \dots, dt^{max}_{k'}\}$ are determined, where $dt^{max}_{k'}$ is the difference of the maximum and the minimum on-times for the k' type ACs. Further, the minimum

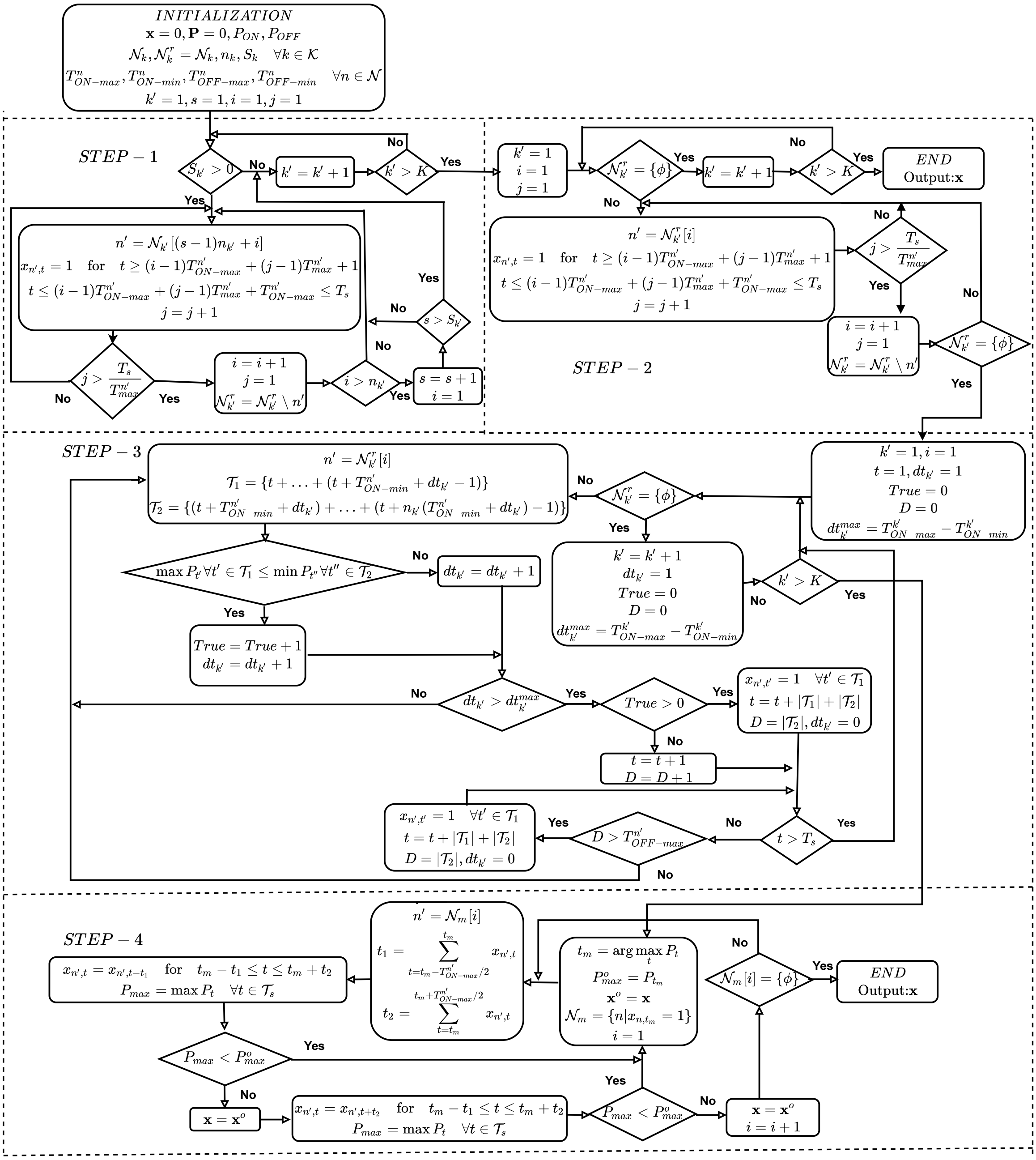


Fig. 2. Block diagram of the proposed heuristic algorithm for ON–OFF scheduling of ACs.

power consumption value for each of the corresponding OFF time windows are determined, i.e., the minimum power consumption are determined for the time windows $[t + T_{ON-min}^{n'} + dt_{k'}, t + n_{k'}T_{ON-min}^{n'} - 1]$, $dt_{k'} = \{0, 1, 2, \ldots, dt_{k'}^{max}\}$. Each of the ON time windows is identified if the maximum power consumption of an ON time window is lower than the minimum power consumption of the corresponding OFF time window. Then, from the identified ON time windows, the ON time window with the largest duration (say, $[t, t+T'-1]$) is chosen to turn ON the compressor of the AC n' of the type k'. Thus, we set $x_{n',t'} = 1, \forall t' \in \{t, t+1, \ldots, t+T'-1\}$. Then, the compressor is kept OFF for the time window $[t+T', t+n_{k'}T'-1]$ and the time slot t is updated as $t = t+n_{k'}T'$. To record the OFF time duration, a variable named as delay D is used. Hence, D is set to $(n_{k'}-1)T'$ that is the duration of the OFF time. However, if any ON time window is not identified satisfying the condition, then $x_{n',t}$ is set as $x_{n',t} = 0$, the time slot t is updated as $t = t + 1$ and delay D is updated as $D = D + 1$. For the new value of t, the above process is repeated and the process is continued, i.e., the compressor of the AC n' is scheduled to be turned ON, and t and D are updated until the delay variable D crosses $T_{OFF-max}^{n'}$. Whenever the delay variable D crosses $T_{OFF-max}^{n'}$, the compressor of the AC n' is turned ON for a duration $T_{ON-max}^{n'}$ and then, $x_{n',t}$, t and D are updated accordingly. The above process is continued until $t \geq T_s$.

3.2.4. Step-4

With completion of Step-3, the scheduling of all the ACs are completed. In Step-4, we propose to shift a number of ON time windows for a number ACs for further reducing the peak load. In this step, the time slot of the maximum power consumption P_{max}, t_m and the set of ACs which are ON in the time slot t_m, $\mathcal{N}_m$ are determined. The ON time windows including the time slot t_m (denoted as $[t_m - t_1, t_m + t_2]$ for an AC $n' \in \mathcal{N}_m$) for the ACs

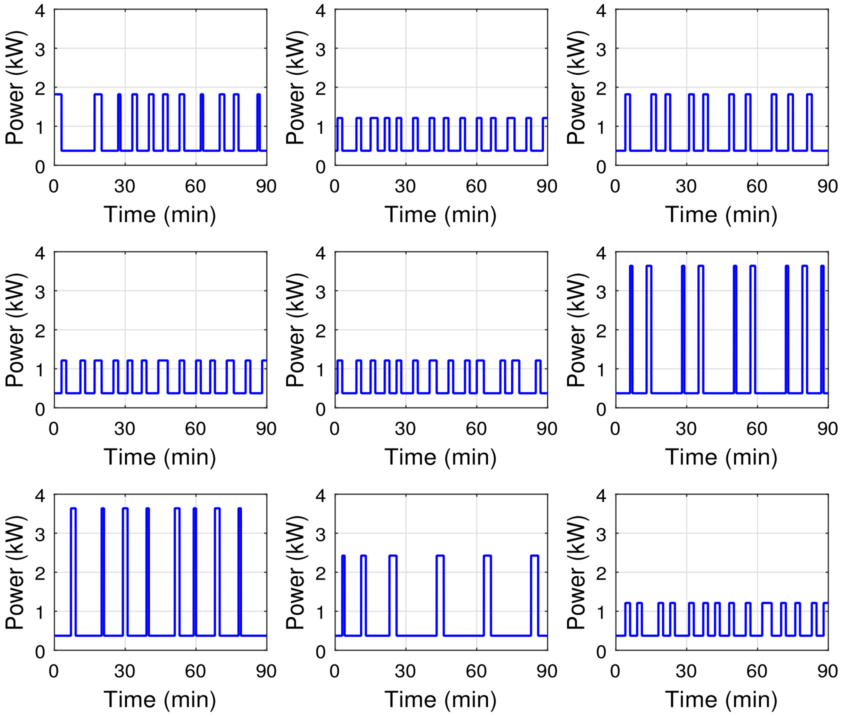


Fig. 3. Power consumptions of the 9 ACs under the optimal coordinated scheduling.

in $\mathcal{N}_m$ are shifted right and then left beyond/before the time slot t_m one after another and the new maximum power consumption is calculated and compared with the previous maximum power consumption until the new maximum power consumption is lower than the previous maximum power consumption. Whenever the new maximum power consumption is found to be lower than the previous maximum power consumption, the load of the AC $n' \in \mathcal{N}_m$ is shifted to the on time window providing new maximum power consumption. Then, the new time slot providing the maximum power P_{max}, t_m is determined again and ON time window shifting process is continued as mentioned above. However, ON time window shifting process as well the heuristic algorithm ends if the new maximum power consumption is found to be higher than or equal to the previous maximum power consumption for all the possible shifting time windows of the ACs in $\mathcal{N}_m$.

4. Results

4.1. Air-conditioning system and parameters

We assume that there are four categories of rooms with areas 30 m^2, 40 m^2, 50 m^2 and 55 m^2 exist in the AC system. Air-conditioners of capacities 1 ton, 1.5 ton, 2 ton and 3 ton are installed to the rooms with areas 30 m^2, 40 m^2, 50m^2 and 55 m^2, respectively. The capacities of the ACs are determined from online available data in [32]. The detail calculations are given in Table 1, where BTU_0, BTU_h, BTU_{ex}, BTU_{Occ}, and BTU_{total} are the BTU/hr for the area with 1 or 2 occupants if the room height is 8 ft, additional BTU/hr for an extra 2 ft height (i.e., we assume that the height of the rooms is 10 ft), extra 10% BTU/hr for sunny region, BTU/hr for extra occupancy and total BTU/hr, respectively. The value of BTU_0 is taken based on the room area, BTU/hr for an extra occupant is taken to be 600 and BTU/hr for extra 1 ft height of an area of 30 m^2 is considered to be 1000. We assume that the number of extra occupants are 1, 2, 3 and 19 for the room areas 30 m^2, 40 m^2, 50 m^2 and 55 m^2, respectively. The room of area 55 m^2 is assumed as drawing/gathering room which has significantly higher number of occupants. From the total BTU/hr, the capacities of ACs are calculated as per relation between BTU/hr and capacity.

Table 1
Calculation of AC size.

Area (m^2)	BTU_0	BTU_h	BTU_{ex}	BTU_{Occ}	BTU_{total}	C (ton)
30	8000	2000	1000	600	11600	1
40	10000	2667	1267	1200	15134	1.5
50	12000	3333	1533	1800	18666	2
55	14000	3667	1767	11400	30834	3

The sizes of ACs could be different for different rooms even when the areas of the rooms are the same. However, for simplicity, we assume that the capacities of ACs for the rooms of same area are equal. Further, we assume that all the rooms have the same ambient temperature and thermal resistance. There are established thermal comfort criteria in international standards and according to the criteria the thermal comfort ranges of human being are approximately 19 °C–25 °C and 22 °C–27 °C during summer and winter, respectively [33]. However, the temperature set point also depends on country, dress up and culture. Thus, the temperature set point H^n_{set} of an AC $n \in \mathcal{N}$ is selected randomly from 16–28 °C as in Bangladesh the consumers set the ACs within this temperature range. The values of H_{out}, η, α, and R_{eq} are taken to be 32 °C, 2.9, 466.150, and 0.35, respectively [29]. Power consumption of the fan of the ACs, P_{fan} is taken to be 373 watt. Note that on-time as well as off-time durations of two ACs of same type may be different due to different temperature set points. However, they become equal due to rounding operation. Thus, the type of ACs is taken to be $K = 4$. For an AC system with N ACs, the number of ACs in each type is randomly selected by using uniform distribution.

4.2. Traditional operation of an air-conditioning system

For traditional operation, the value of ΔH is considered to be 4 °C which provides on-time and off-time durations for each of the ACs. The ACs operate independently. We assume that an AC

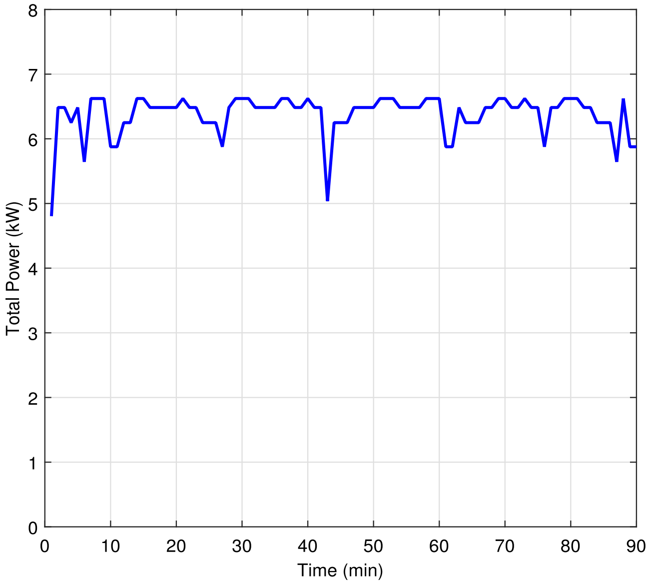


Fig. 4. Total power consumptions for the 9 ACs under the optimal coordinated scheduling.

$n \in \mathcal{N}$ starts, i.e., turns ON its compressor at time t_n^s and the time t_n^s is selected randomly using uniform random distribution between zero to off-time duration. The AC n then turns OFF its compressor after on-time duration, turns ON its compressor after off-time duration and so on.

4.3. Air-conditioning system operation by the optimal and heuristic solutions

For the optimal and heuristic solution based operations, we consider $\Delta H_l = 2$ and $\Delta H_u = 6$. Using the dead-band range, AC capacity and the other parameters, the values of P_{ON}^n, T_{ON-max}^n, T_{ON-min}^n, $T_{OFF-max}^n$, and $T_{OFF-min}^n$ are calculated for all $n \in \mathcal{N}$. We consider a time slot equal to one minute and hence, T_{ON-max}^n, T_{ON-min}^n, $T_{OFF-max}^n$, are $T_{OFF-min}^n$ are modified as integers by rounding operation. The optimization problem is solved using the optimal and heuristic solution approaches and power consumption at each time slot is determined considering $T_s = 90$ min.

4.4. Optimal coordinated scheduling

We solve the optimization problem for 9 ACs. The capacities of the 9 ACs are found to be 1.5 ton, 1 ton, 1.5 ton, 1 ton, 1 ton, 3 ton, 3 ton, 2 ton and 1 ton from random selection. The power consumption values of the 9 ACs at the different time instants under the optimal coordinated scheduling are presented in Fig. 3. It can be observed that the optimal scheduling tries to turn on less number of ACs simultaneously such that the peak load is minimized. The total power consumption of the 9 ACs is shown in Fig. 4. The results show that the optimal solution minimizes the peak load as well as the variation of the total power consumption at the different time instants.

4.5. Advantages of the optimal coordinated scheduling

For a given number of ACs, we generate 20 instances of air-conditioning system for averaging the randomness in the number of different types of AC and the temperature set point. For the same 20 instances, we generate results under the traditional, optimal and heuristic approaches and take the average values.

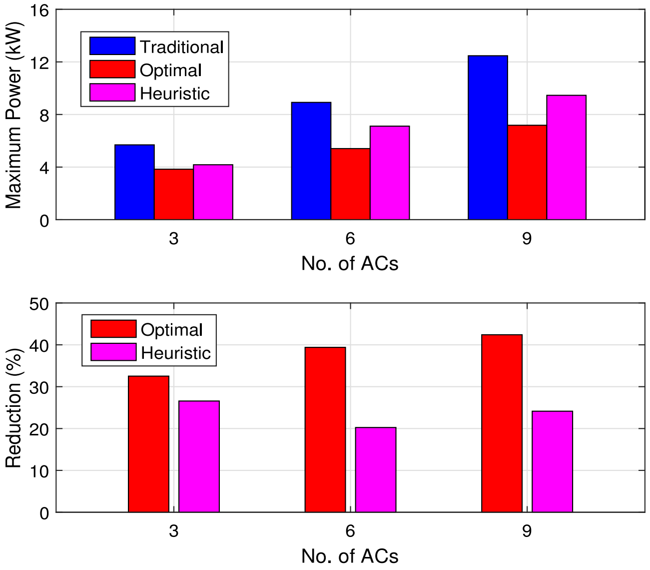


Fig. 5. Maximum power consumption under different approaches.

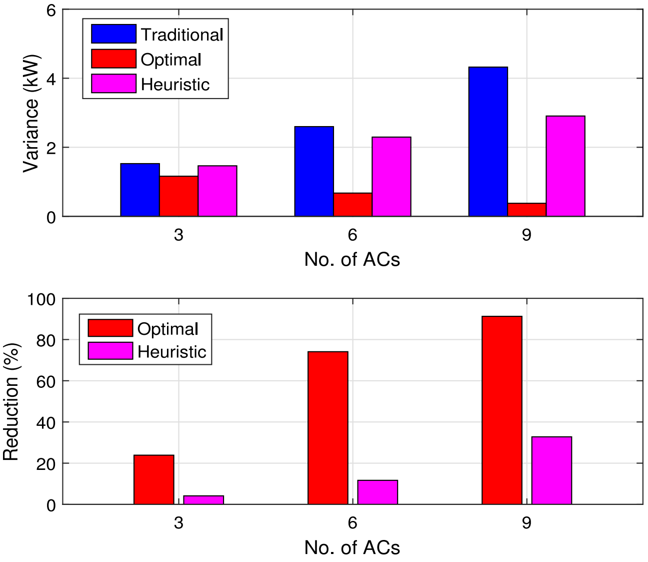


Fig. 6. Variance of power consumption under different approaches.

The traditional and the heuristic approaches are implemented by using MATLAB tool run in a CPU of 2.67 GHz processor and 4 GB RAM. The maximum power consumption obtained under the traditional, optimal and heuristic approaches is presented in Fig. 5. Note that we are able to solve the problem optimally up to $N = 9$ within the time limit of the NEOS solver. The results show that the peak load increases with increment of the number of ACs in the system due to the increment of total load. We find that the optimal solution approach reduces the peak load significantly and the reduction is in the range of 33%–42%. Although the reduction of the peak load by the heuristic approach is lower than the optimal one, the reduction is found to be more than 20% for an AC system up to 9 ACs. Thus, coordinated ON–OFF scheduling of the compressors of ACs can reduce the peak load keeping the temperature set point of the consumers unchanged, i.e., without providing discomfort to the consumers.

We determine the variance of power consumption at different times/time-slots. The results on load variance are presented in Fig. 6. The results show that the optimal approach reduces the variance of load remarkably. Further, the reduction of variance

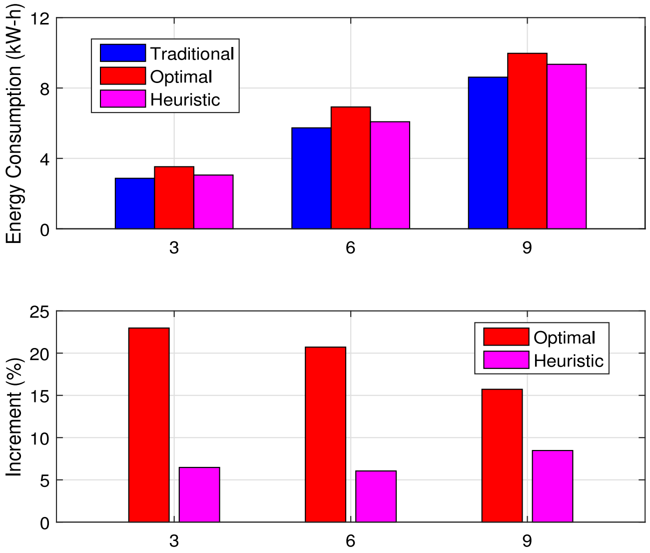


Fig. 7. Energy consumption under different approaches.

Table 2
Computation time of the optimal and heuristic solution approaches.

No. of ACs	Optimal (ms)	Heuristic (ms)
3	150	0.0045
6	121250	0.0153
9	9040000	0.0262

of load by the heuristic approach is also significant which is about 60% for a higher number of ACs. Thus, the stability of the connected grid can be enhanced by reducing the peak load and the variance of load with the proposed approaches. Further, we determine the total energy consumption for the time duration 90 min. The results on energy consumption are presented in Fig. 7. The results show that the energy consumption increases under the proposed approaches. Thus, there is a trade-off between peak load reduction and energy consumption. However, the percentage of increment of energy consumption decreases with increasing the number of ACs and it might be 10%–15%.

4.6. Advantages of the heuristic solution approach

The main limitation of the optimal solution based approach is the computation complexity. To overcome the computation complexity, the heuristic solution approach is proposed. We present the computation time of the optimal and heuristic solution approaches in Table 2. The computation times for optimal solution are 0.15 s, 2.02 min and 150.67 min for the number of ACs 3, 6 and 9, respectively. Thus, practical implementation of the optimal solution approach is infeasible due to the computation time limitation. On the other hand, the computation time of the heuristic solution approach is very low, i.e., less than one second for the number of ACs 3, 6 and 9. Thus, the heuristic approach could be a better approach for practical implementation.

Since the computation time of the heuristic solution is very less, to evaluate the performance of the heuristic approach in case of an AC system with higher number ACs, we determine the 90 min scheduling of AC compressors for large size AC systems with different number of ACs. For averaging the randomness in the number of different types of AC and the temperature set point, we take 500 instants for a given number of ACs. The maximum power, variance of power consumption and energy consumption results are presented in Fig. 8, Fig. 9, and Fig. 10, respectively. We find that the heuristic approach based compressor scheduling can reduce the peak load and load variance about 15% and 50%, respectively with 1%–2% increment of energy consumption compared to the traditional operation of AC system. We also present the computation time of the heuristic solution approach in Fig. 11 for different number of ACs. Although the computation time increases with the number of ACs, they are in the scale of millisecond, i.e., they are significantly less compared to 90 min. Thus, heuristic solution approach is applicable for an AC system with large number of ACs.

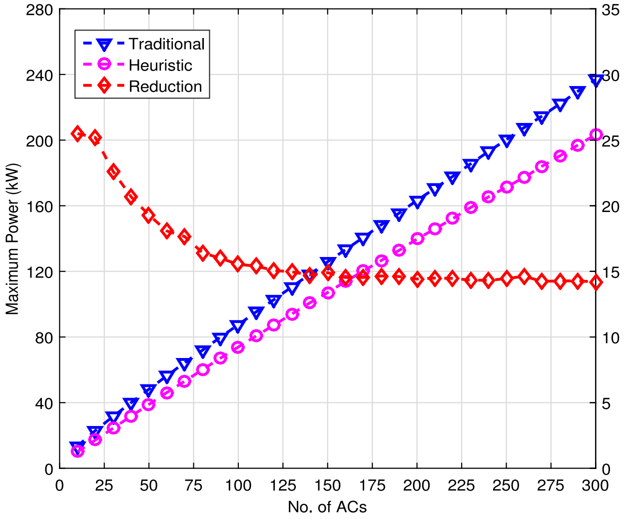


Fig. 8. Maximum power consumption under different approaches.

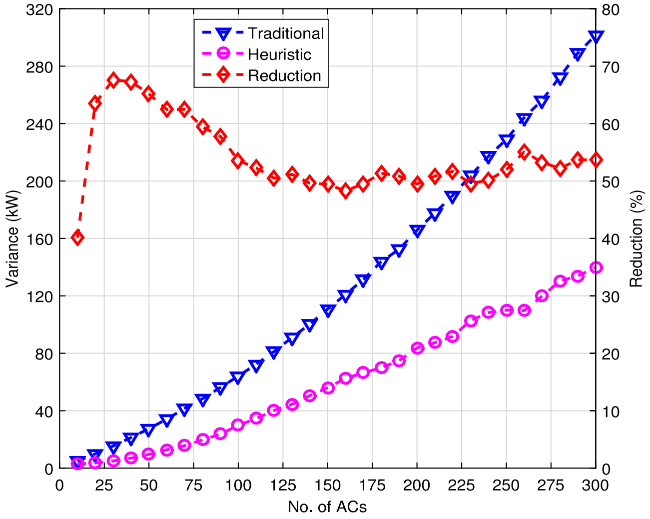


Fig. 9. Variance of power consumption under different approaches.

5. Other issues and possible extensions of the work

In this paper, we consider the simple models of on-time and off-time durations considering the ambient temperature as depending variable. Using the simple models, the optimization problem is formulated and solved to minimize the peak load in a time duration. The same problem needs to be solved for next time intervals with the new values of ambient temperature. Note that the effects of solar gain, internal gain and building/zone dynamics are not studied in this paper. The on-time and off-time durations of an AC also depend on these variables. Since the optimization

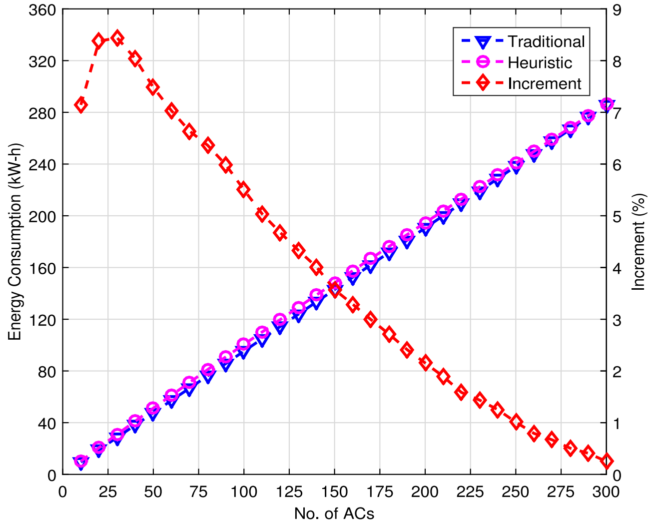


Fig. 10. Energy consumption under different approaches.

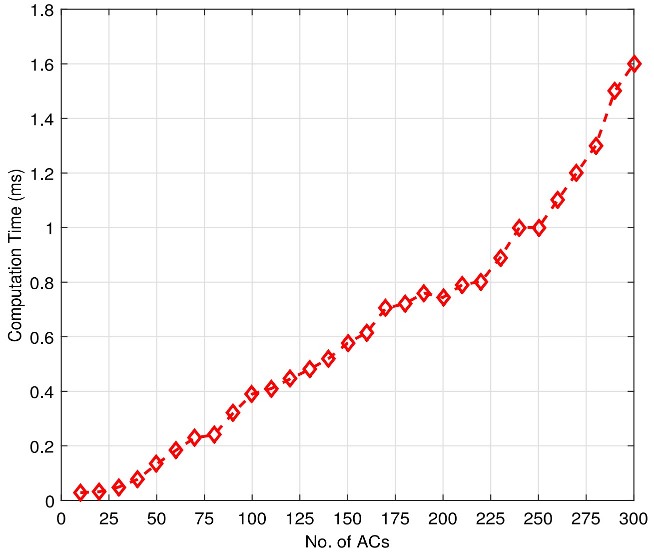


Fig. 11. Computation time of the heuristic solution.

problem is valid for any on-time and off-time durations of the ACs in the system, one can include the effects of the other parameters on on-time and off-time durations and solve the optimization problem. Intuitively, it can be said that the large variations of on-time and off-time durations for considering the other effects will provide more flexibility to schedule the AC compressors and the reduction of the peak load will be more by optimal scheduling.

The proposed coordinated scheduling reduces the (instantaneous) peak load during grid peak and off-peak periods. We assume that consumer comfort level is 100% if the temperature set point can be kept fixed to the consumer's temperature set point and the optimization problem formulated to provide 100% comfort level. The most of the proposed existing research works [16–27] on the reduction of peak load by AC scheduling consider a range of temperature set points for comfort level of the consumers and increases the temperature set point of the ACs during the grid peak to reduce the peak load. The proposed optimization framework can be extended to reduce the peak load by integrating the control of temperature set points of the consumers and coordinated ON–OFF scheduling of AC compressors, and hence, the peak load reduction will further increase significantly.

In this work, the non flexible loads, e.g., loads for light, fan, communicating and computing devices and equipment are not included in the optimization problem. The nature of the non-flexible loads is not similar to the air-conditioner loads and the non-flexible load cannot be scheduled. In practice, the non-flexible load at a time *t* cannot be known prior to the time *t* and hence, the problem needs to be addressed in a different way if the non-flexible load needs to be included. Further, we assume that the ambient temperature and the coefficient of performance remain unchanged during the 90 min duration. However, there may be little variations in the ambient temperature and the coefficient of performance within the 90 min duration. According to our proposal, the compressors of the air-conditioners will run according to the optimal on–off scheduling even when there are variations in the ambient temperature and the coefficient of performance during the 90 min duration and hence, a little deviation may observe on the bounds of the dead-band temperature. Moreover, many practical issues, e.g., frequency regulation due to variable-speed compressor operation, embedded, on–off control and other control and communication system design, need to be solved to implement the idea of coordinated ON–OFF scheduling of the compressors of ACs. All of the research issues mentioned above remain open for future studies.

6. Conclusion

We have introduced the notion of coordinated ON–OFF scheduling of compressors of ACs of an air-conditioning system to reduce the peak load and load variance without changing the temperature set point and comfort of the consumers. We have modelled the necessary constraints for operation of ACs and formulated an optimization problem to minimize peak load. We have optimally solved the problem using CPLEX solver of NEOS server. However, due to the complexity of the problem, a long time is required to solve the problem and the problem is not tractable for a large number of ACs. We have found that the optimal approach significantly reduces the peak load as well as load variance with some increment of energy consumption compared to a traditional operation of air-conditioning system. We have also developed a heuristic solution approach which solves the problem within very short time and reduces peak load and load variance significantly.

CRediT authorship contribution statement

Md. Forkan Uddin: Conception and design of study, Analysis and/or interpretation of data, Writing – original draft, Writing – review & editing. **K M Naimul Hassan:** Acquisition of data, Analysis and/or interpretation of data, Writing – original draft. **Soumav Biswas:** Acquisition of data, Analysis and/or interpretation of data, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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